Modeling, Verification and Applications of Explicit Time Models

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ANR Project DOTS

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Verification is necessary

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for critical systems

Classical verification problems

- reachability of a control state
- $\mathcal{S} \sim \mathcal{S}'$ bisimulation, etc.
- $L(\mathcal{S}) \subseteq L(\mathcal{S}')$ language inclusion
- $\mathcal{S} \models \varphi$ for some formula φ model-checking
- reachability on $\mathcal{S} \parallel A_T$, product of $\mathcal{S}$ with testing automaton A_T
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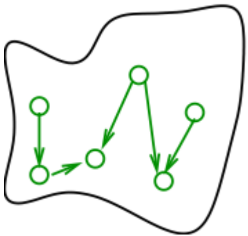
Modeling



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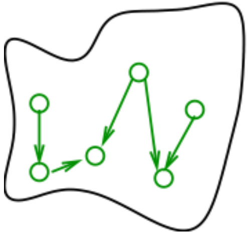
Does the system meet its specification ?

Modeling _ _ _ _ _ _ _ _ _

$\mathcal{S}$



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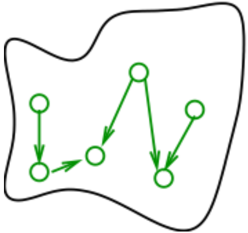

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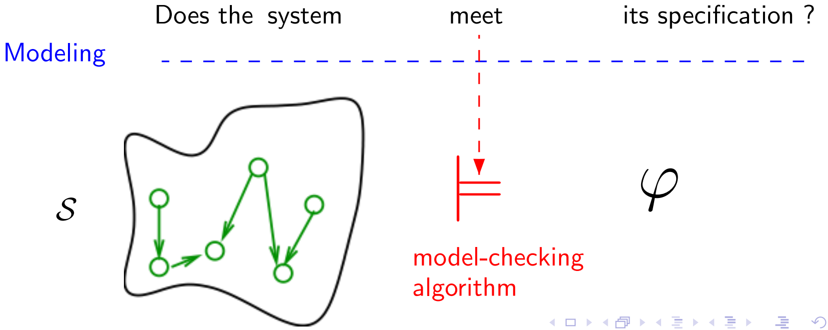
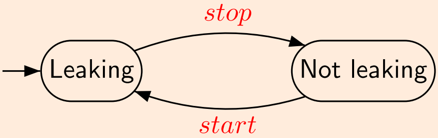


Why add time ?

The gas burner example [ACHH93]

The gas burner may leak and :

- each time a leakage is detected, it is repaired or stopped in less than 1s
- two leakages are separated by at least 30s



Is it possible that the gas burner leaks during a time greater than $\frac{1}{20}$ of the global time after the first 60s?

Timed features are needed in the model and in the properties:

Instead of observing a sequence of events $a_1 a_2 \ldots$,
observe a sequence of pairs $(a_1, t_1)(a_2, t_2)\ldots$ where t_i is the time at which a_i occurs.

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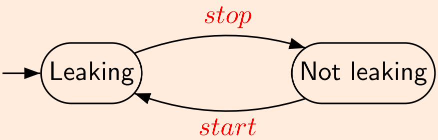

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Outline

Timed Models

Verification

Applications

Conclusion

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Transition systems

Definition

$\color{red}{Act}$ alphabet of actions

$\mathcal{T} = (S, s_0, E)$ transition system

- S set of configurations, s_0 initial configuration,
- $E \subseteq S \times Act \times S$ contains

  action transitions: $s \xrightarrow{a} s'$, instantaneous execution of a

Example: a finite automaton

An execution:

ok $\xrightarrow{p}$ fault $\xrightarrow{r}$ ok $\xrightarrow{p}$ fault $\xrightarrow{h}$ alarm $\xrightarrow{r}$ ok $\longrightarrow \cdots$

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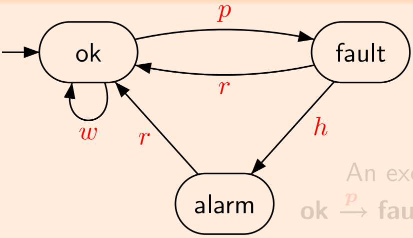

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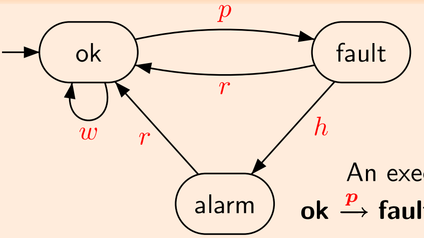

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Timed Transition Systems

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  delay transitions: $s \xrightarrow{d} s'$, time elapsing for d time units.

Timed Transition Systems

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$\mathcal{T} = (S, s_0, L, E)$ timed transition system

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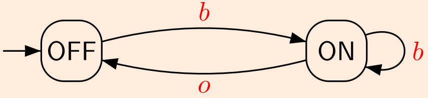
Why not discretize ?

A time switch



b button pressed

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Unfolding with discrete time

when adding the constraint: the light stays on exactly 3 time units once the button is pressed.

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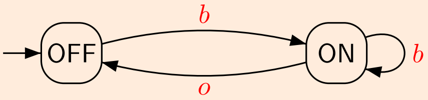

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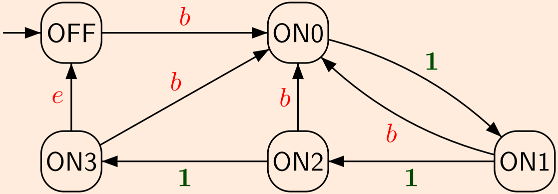

$\mathbf{1}$ wait for 1 t.u.

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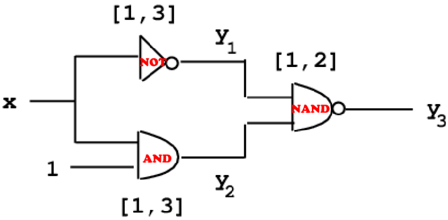
Discussion: reachable configurations

for asynchronous digital circuits [Alur 1991] [Brzozowski Seger 1991]



Start with x=0 and y=[101] (stable configuration)

Input x changes to 1. The corresponding stable configuration is y=[011]

However, many possible behaviours, e.g.

$$[101] \xrightarrow[1.2]{y_2} [111] \xrightarrow[2.5]{y_3} [110] \xrightarrow[2.8]{y_1} [010] \xrightarrow[4.5]{y_3} [011]$$

Reachable configurations: {[101], [111], [110], [010], [011], [001]}

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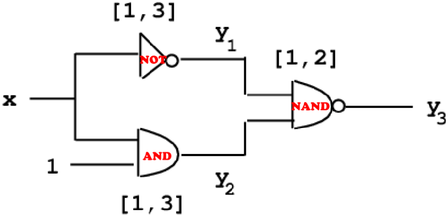

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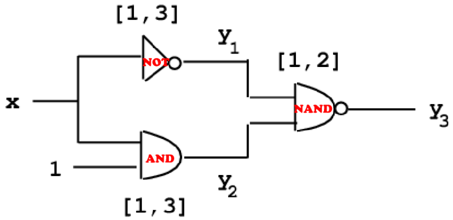

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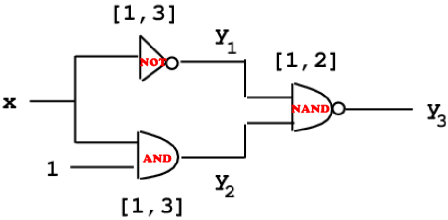

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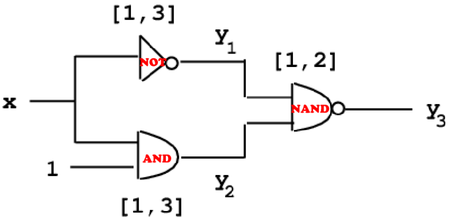

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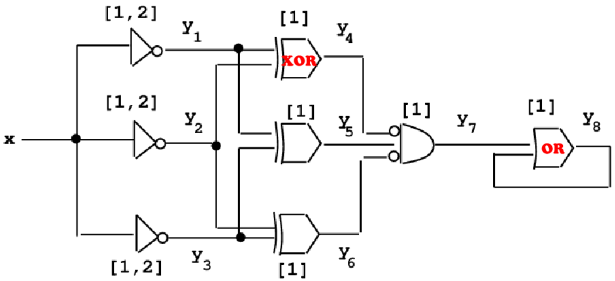
A circuit which is not 1-discretizable



Why? initially $x = 0$ and $y = [11100000]$, then x is set to 1

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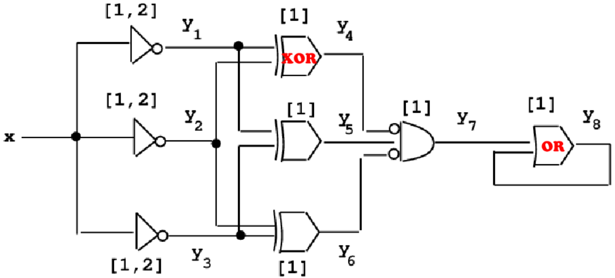

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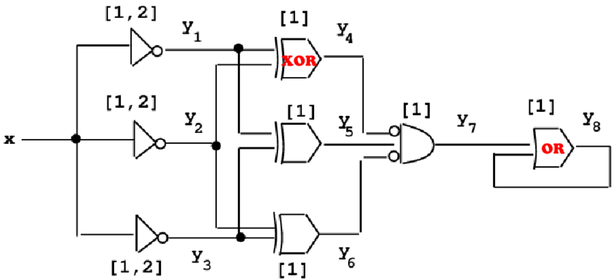

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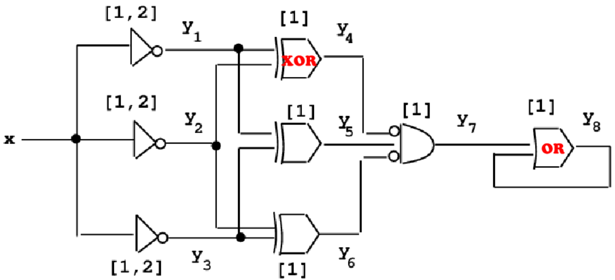

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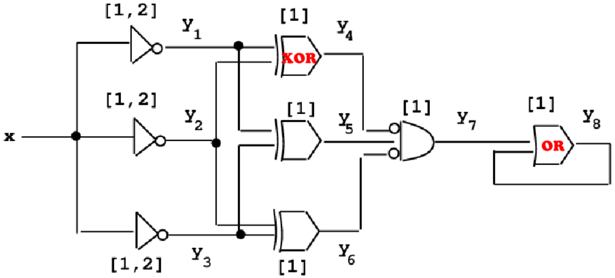

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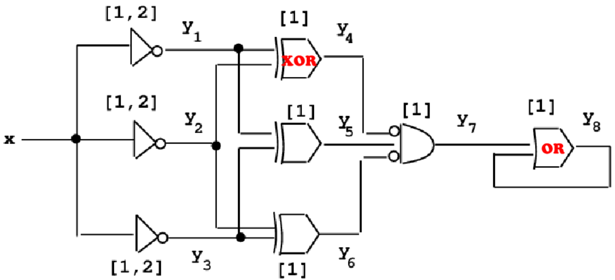

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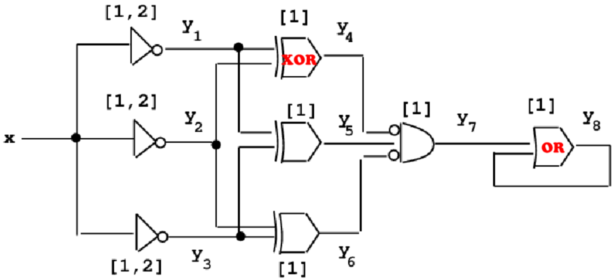

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Is discretizing sufficient?

Theorem [Brzozowski Seger 1991]

For every $k \geq 1$, there exists a circuit such that the set of reachable states is strictly larger in dense time than in discrete time (with granularity $\frac{1}{k}$).

Consequence

Finding a correct granularity may be as difficult as computing the set of reachable states in dense-time

Furthermore

there exist systems for which no discrete execution is possible, whatever the granularity choice.

(see later)

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Adding time intervals on transitions (1)

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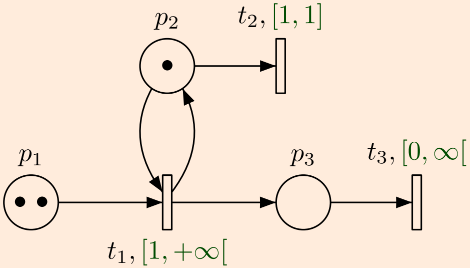

Markings: $M_0 = (2, 1, 0)$, $M_1 = (1, 1, 1)$, $M_2 = (0, 1, 2)$, $M_3 = (0, 0, 2)$
Time valuation of a transition t: time since t was last enabled, $\perp$ if t is not enabled.

An execution:
$(M_0, [0, 0, \perp]) \xrightarrow{1} (M_0, [1, 1, \perp]) \xrightarrow{t_1} (M_1, [1, 1, 0]) \xrightarrow{t_1} (M_2, [\perp, 1, 0]) \xrightarrow{t_2}$
$(M_3, [\perp, \perp, 0]) \xrightarrow{1.5} (M_3, [\perp, \perp, 1.5]) \cdots$

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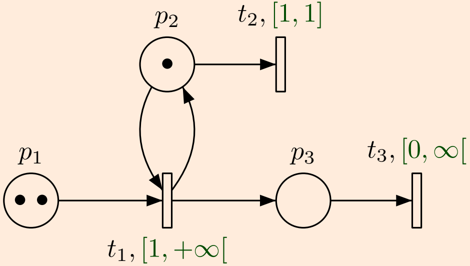

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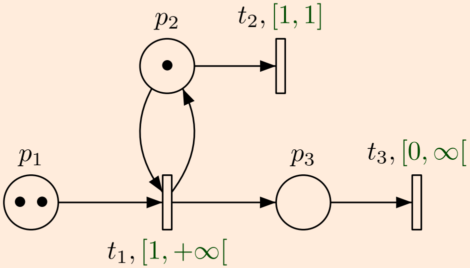

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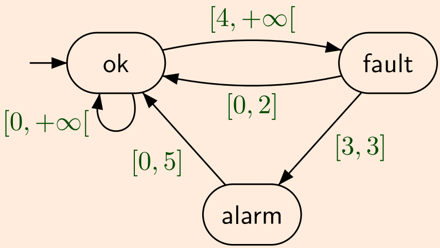
Time valuation of a transition t: time since t was last enabled, $\perp$ if t is not enabled.

An execution:

$(\boldsymbol{M_0}, [\mathbf{0}, \mathbf{0}, \perp]) \xrightarrow{\mathbf{1}} (\boldsymbol{M_0}, [\mathbf{1}, \mathbf{1}, \perp]) \xrightarrow{\boldsymbol{t_1}} (\boldsymbol{M_1}, [\mathbf{1}, \mathbf{1}, \mathbf{0}]) \xrightarrow{\boldsymbol{t_1}} (\boldsymbol{M_2}, [\perp, \mathbf{1}, \mathbf{0}]) \xrightarrow{\boldsymbol{t_2}}$
$(\boldsymbol{M_3}, [\perp, \perp, \mathbf{0}]) \xrightarrow{\mathbf{1.5}} (\boldsymbol{M_3}, [\perp, \perp, \mathbf{1.5}]) \cdots$

Adding time intervals on transitions (2)

Example 2: finite automata with delays [Emerson et al. 1992]



An execution: **ok** $\xrightarrow{15}$ **fault** $\xrightarrow{1.5}$ **ok** $\xrightarrow{8}$ **fault** $\xrightarrow{3}$ q_2 $\xrightarrow{2.7}$ **ok** $\cdots$

Remark: only delay transitions

Adding time intervals on transitions (2)

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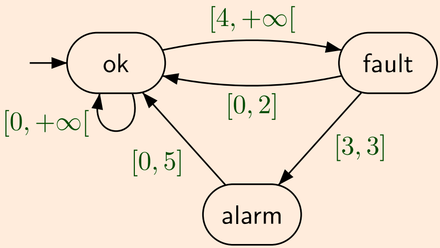

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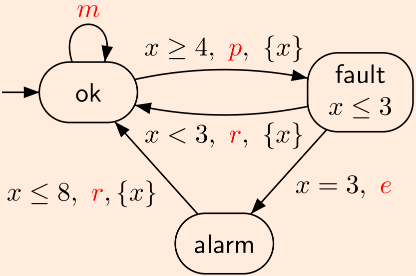
Adding clocks: timed automata (1)

A variation of [Alur Dill 1990]



$\boldsymbol{x}$ real valued clock

$\boldsymbol{x < 3}$, $\boldsymbol{x = 3}$, $\boldsymbol{x \geq 4}$ guards

$\boldsymbol{x \leq 3}$ invariant

$\boldsymbol{\{x\}}$ reset operation for $\boldsymbol{x}$

also written $\boldsymbol{x := 0}$

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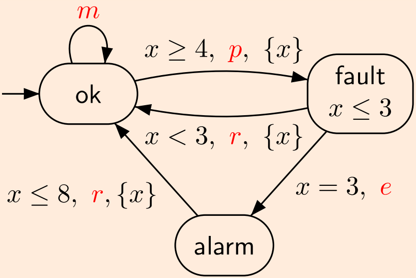

x real valued clock

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Clock valuations and clock constraints

X a set of clocks, valuation $v : X \mapsto \mathbb{R}_{\geq 0}$,
$\mathcal{C}(X)$ set of clock constraints: conjunctions of atomic constraints of the form $x \bowtie c$, for clock x, constant c and $\bowtie$ in $\{<, \leq, =, \geq, >\}$.

Adding clocks: timed automata (1)

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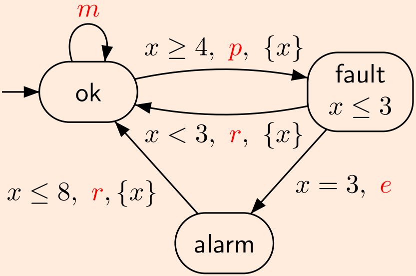

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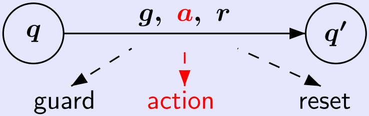
Timed automaton $\mathcal{A} = (Q, q_0, Inv, \Delta)$

- Q set of (control) states, q_0 initial state,
- Inv associates an invariant with each state
- Δ contains transitions :



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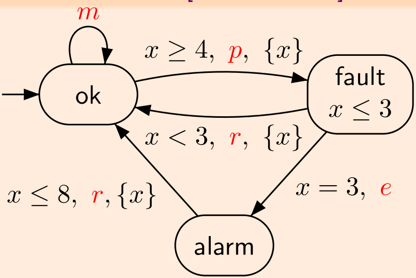

An execution: $(\mathbf{ok}, [0]) \xrightarrow{8.3} (\mathbf{ok}, [8.3]) \xrightarrow{p} (\mathbf{fault}, [0]) \xrightarrow{3} (\mathbf{fault}, [3]) \xrightarrow{e} (\mathbf{alarm}, [3]) \xrightarrow{2.1} (\mathbf{alarm}, [5.1]) \xrightarrow{r} (\mathbf{ok}, [0]) \cdots$

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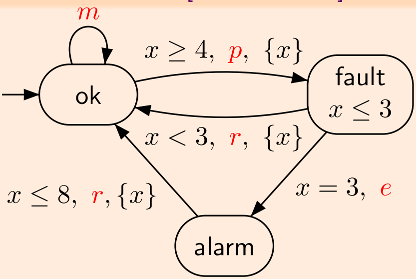

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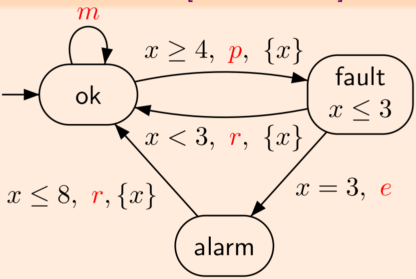

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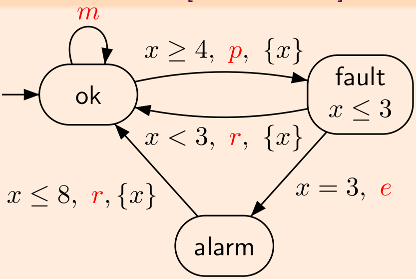

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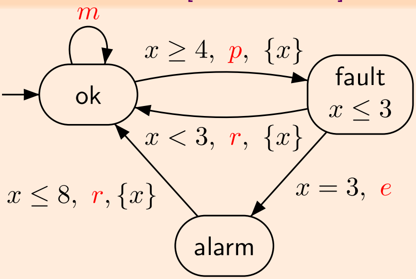

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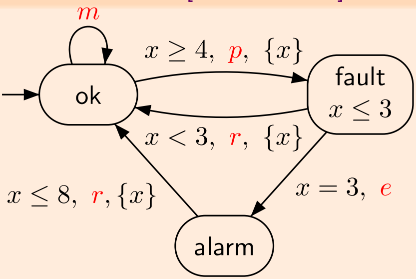

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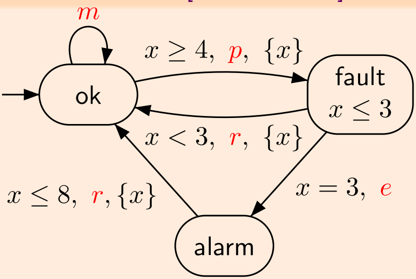

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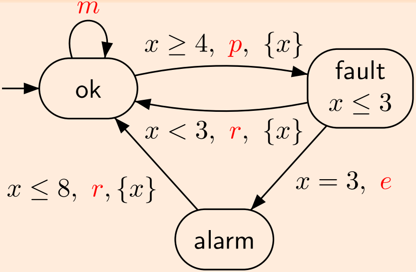

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Semantics of timed automata (1)

Operations on valuations

X set of clocks. For valuation v :

- for a subset r of X, valuation $v[r \mapsto 0]$ is obtained by reset of the clocks in r, other values unchanged,
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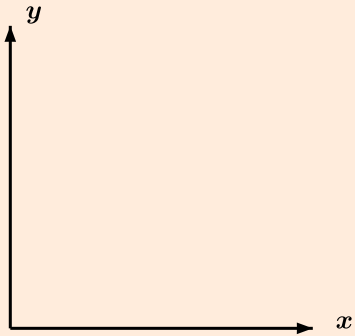
Geometric view with two clocks x et y



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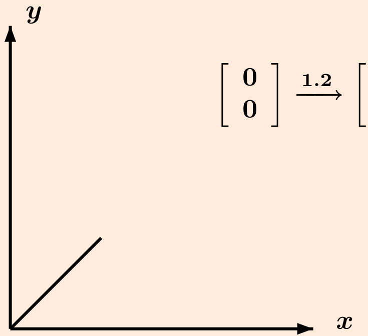

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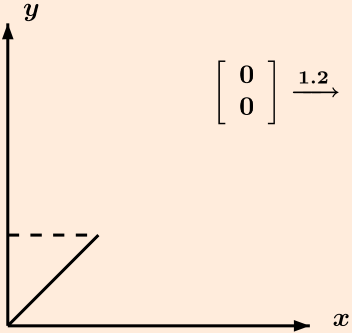

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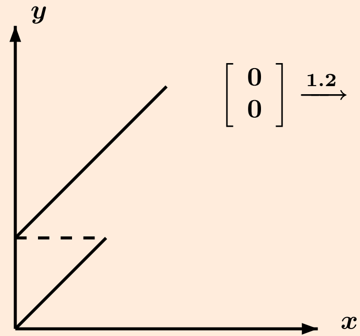

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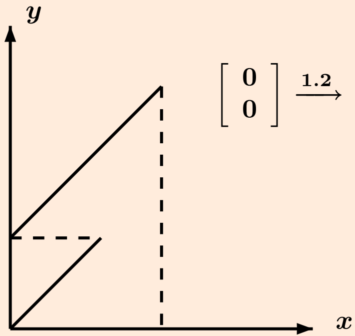

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Semantics of timed automata (2)

Definition

For a timed automaton $\mathcal{A} = (Q, q_0, Inv, \Delta)$, the transition system is $\mathcal{T} = (S, s_0, E)$ with:

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- initial configuration $s_0 = (q_0, \mathbf{0})$,
- action transitions: $\boldsymbol{(q, v) \xrightarrow{a} (q', v')}$, if there exists a transition $q \xrightarrow{g,a,r} q'$ from $\mathcal{A}$ such that $v \models g$ and $v' \models Inv(q')$, with $v' = v[r \mapsto 0]$,
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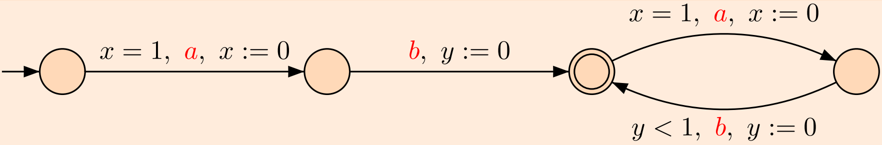
Discrete vs dense time (revisited)

[Alur Dill 1994]



- Dense-time
  The infinite observation $(a,1)(b,2)(a,2)(b,2.9)(a,3)(3.8)(a,4)(b,4.7)\ldots$
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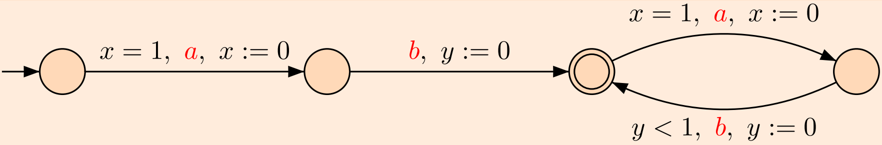

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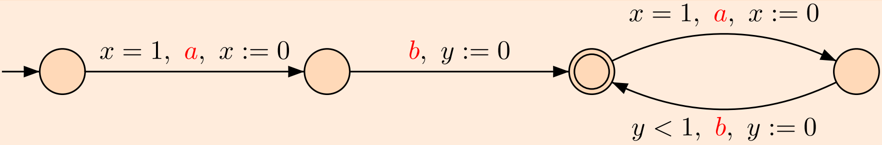

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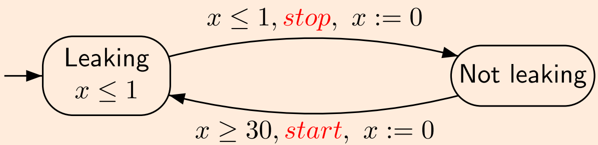
The gas burner (revisited)

as a timed automaton

- each time a leakage is detected, it is repaired or stopped in less than 1s
- two leakages are separated by at least 30s



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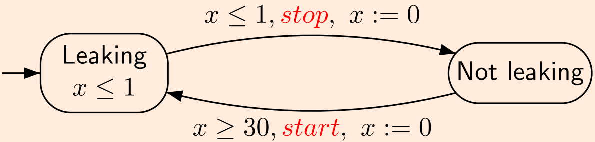

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Timed logics

Temporal logics

A request is always granted

in Computational Tree Logic CTL

$$\mathsf{AG}(\mathsf{request} \Rightarrow \mathsf{AF}\,\mathsf{grant})$$

How to express:

A request is always granted in less than 5 time units

CTL + time: TCTL [Alur Henzinger 1991]

$$\varphi, \psi ::= P \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathsf{E}\varphi\mathsf{U}_{\bowtie c}\psi \mid \mathsf{A}\varphi\mathsf{U}_{\bowtie c}\psi$$

P an atomic proposition, c a constant and $\bowtie$ an operator in $\{<, >, \leq, \geq, =\}$.

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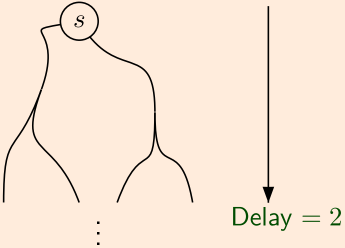
A formula is interpreted on a configuration of a TTS



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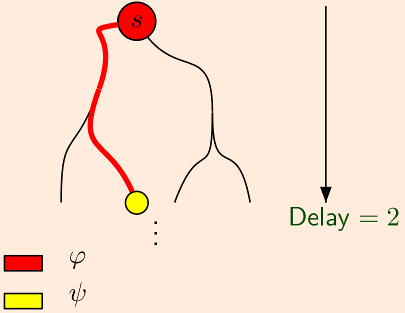

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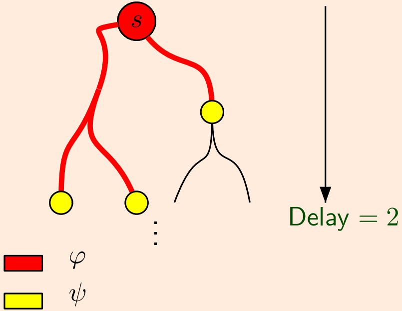

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Interpretation

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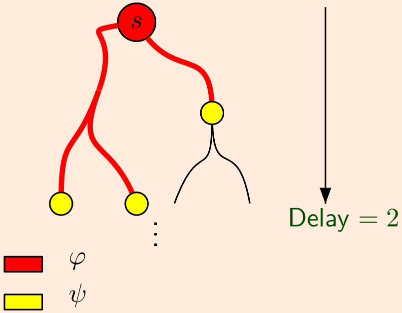


$$s \models \mathsf{A}\varphi\mathsf{U}_{\leq 2}\psi$$

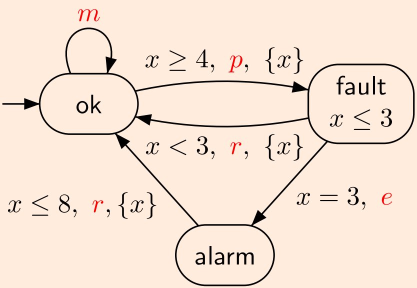
Abbreviations

$\mathsf{AF}_{\bowtie c}\psi$ means $\mathsf{A}\ true\ \mathsf{U}_{\bowtie c}\psi$

$\mathsf{EF}_{\bowtie c}\psi$ means $\mathsf{E}\ true\ \mathsf{U}_{\bowtie c}\psi$

$\mathsf{AG}_{\bowtie c}\psi$ means $\neg\mathsf{EF}_{\bowtie c}(\neg\varphi)$

Example for a timed automaton



initial state ok satisfies:

$$\mathsf{AG}(\mathsf{fault} \Rightarrow \mathsf{AF}_{\leq 8}\, \mathsf{ok})$$

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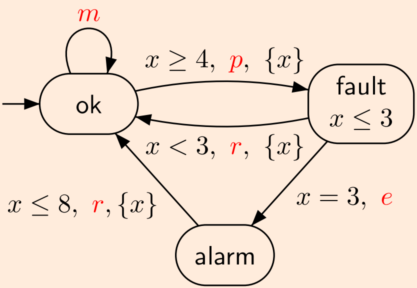

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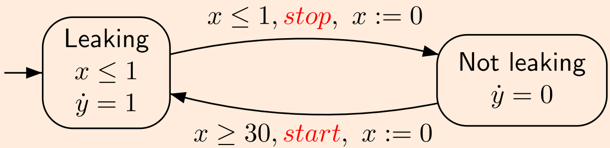
Other logics

Back again to the gas burner

as a linear hybrid automaton



Add a stopwatch y and a clock z which are never reset

and use these variables in a CTL formula:

$$\mathsf{AG}(z \geq 60 \Rightarrow 20y \leq z)$$

Timed logics for linear time

Extensions of Linear Temporal Logic LTL

- with intervals as subscript: MTL, with non singular intervals: MITL,
- with clocks in formulas...

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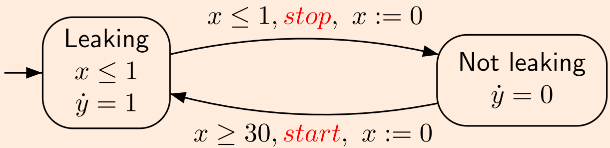

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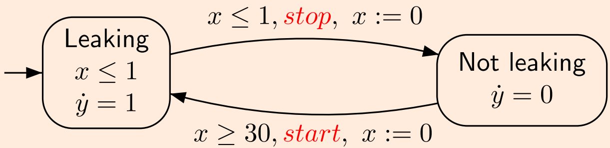

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Verification

Applications

Conclusion

Reachability

Deciding reachability of a control state reduces to decide emptiness.

Theorem [Alur Dill 1990]

The emptiness problem for timed automata is PSPACE-complete.

Decision procedure

Input: a timed automaton $\mathcal{A} = (Q, q_0, Inv, \Delta)$ on a set X of real valued clocks

- Construction of a (Büchi) standard automaton $\mathcal{H}$, such that:
  no execution possible in $\mathcal{A} \Leftrightarrow$ no execution possible in $\mathcal{H}$
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$\mathcal{T} = (S, s_0, E)$

transition system of $\mathcal{A}$

configurations: (q, v)

$q \in Q$, $v \in \mathbb{R}^X_{\geq 0}$

$\xrightarrow{quotient}$

$\mathcal{H}$

region automaton of $\mathcal{A}$

states: $(q, [v])$

$q \in Q$, $[v]$ equivalence class

for some relation $\sim$ on $\mathbb{R}^X_{\geq 0}$

Quotient construction (1)

with the following properties:

For two equivalent valuations $v \sim v'$

1. if an action transition $q \xrightarrow{g,a,r} q'$ is possible from v, then the same transition is possible from v' and the resulting valuations $v[r \mapsto 0]$ et $v'[r \mapsto 0]$ are equivalent,
2. if a delay transition of $\boldsymbol{d}$ is possible from v, then a delay transition of $\boldsymbol{d'}$ is possible from v' and the resulting valuations $v + d$ et $v' + d'$ are equivalent.

Remarks

- Relation $\sim$ produces a time-abstract bisimulation between configurations (q, v) of $\mathcal{T}$ and states $(q, [v])$ of $\mathcal{H}$.
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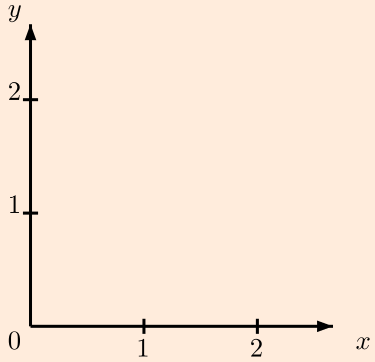
Quotient construction (2)

Geometric view with two clocks x and y, for $m = 2$



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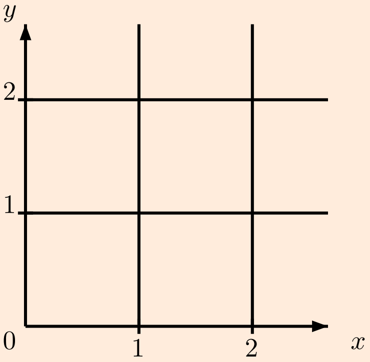

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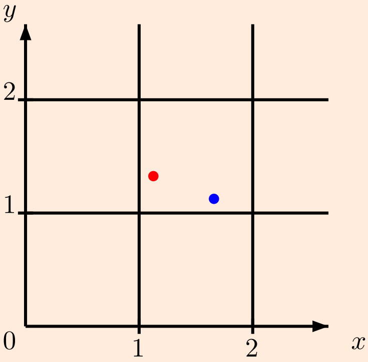

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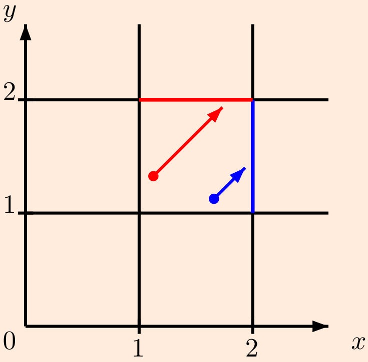

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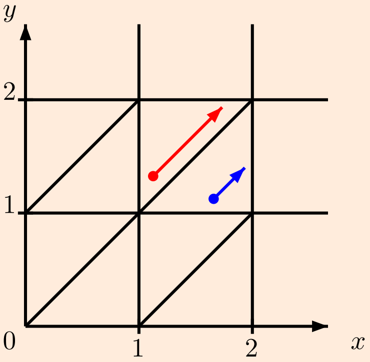

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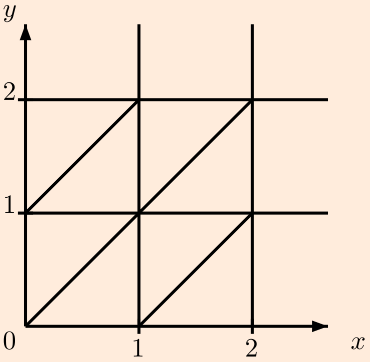

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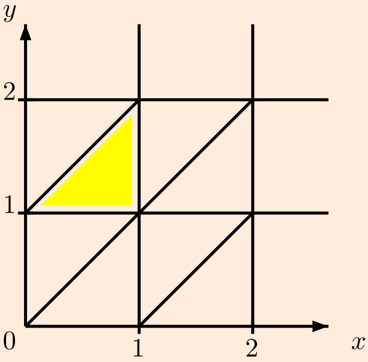

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$I_x =]0;1[$, $I_y =]1;2[$

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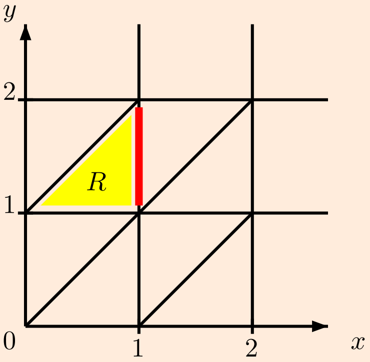

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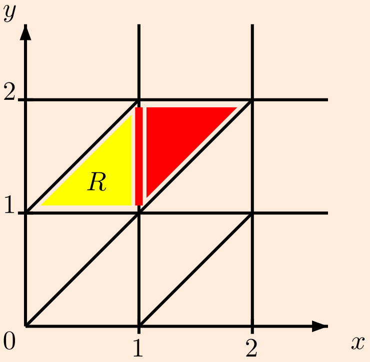

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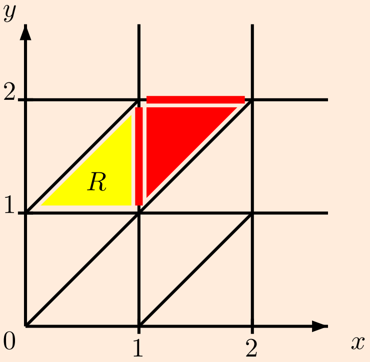

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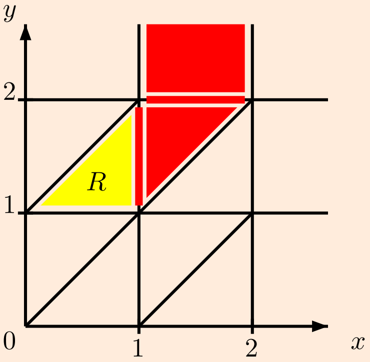

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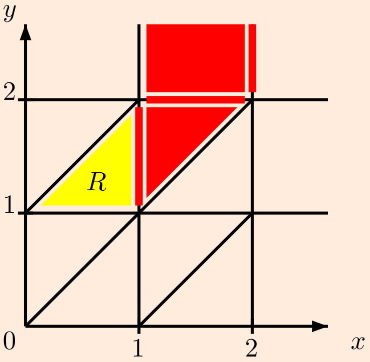

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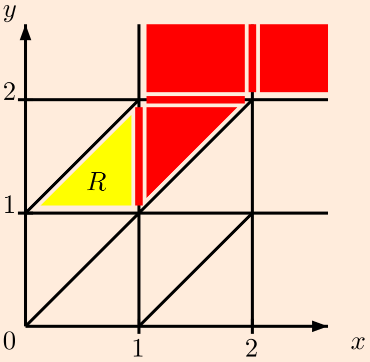

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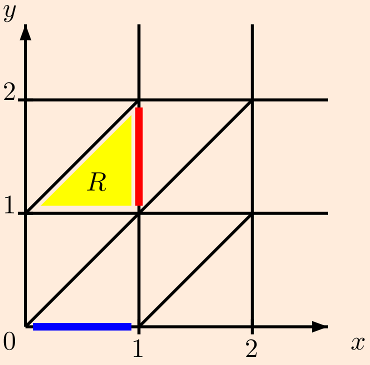

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Time successor of R

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Action successor of R

with $y := 0$

$I_x =]0;1[$, $I_y = [0;0]$

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Region automaton $\mathcal{H}$

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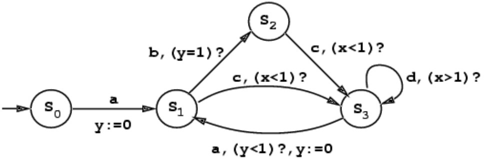

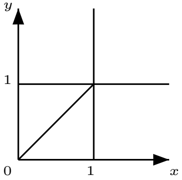

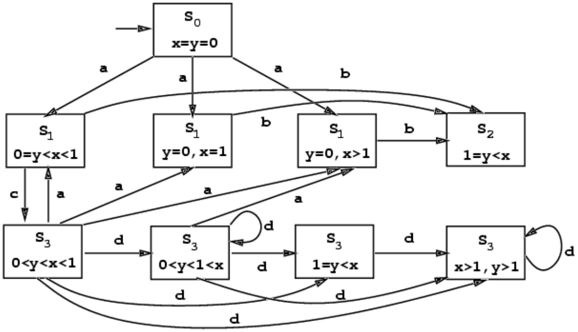

Other results

Complexity is higher than for untimed models

- The model-checking problem for TCTL on timed automata is PSPACE-complete [Alur et al. 1993] .
- The model-checking problem for MITL on timed automata is EXPSPACE-complete [Alur et al. 1996].

and sometimes worse:

The model-checking problem for MTL on timed automata is undecidable [Henzinger 1991].

Some efficient algorithms

by restriction: for the logic $\mathsf{TCTL}_{\leq,\geq}$ (without equality)

- for automata with duration and discrete time, model-checking is in polynomial time ($|\mathcal{A}| \cdot |\varphi|$) [Laroussinie et al. 2002].
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- KRONOS and UPPAAL for timed automata
- HCMC and HYTECH for linear hybrid automata (semi-algorithms)
- TSMV for automata with duration (discrete time)
- Romeo and TINA, for time Petri nets
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using specific data structures

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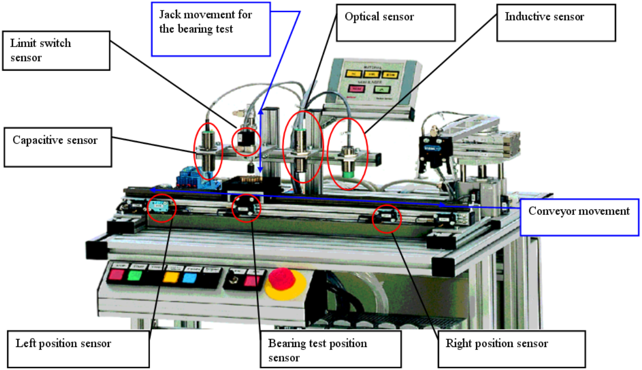

Presentation of MSS station 2

- Work-pieces are transported by a linear conveyor
- They are tested by a jack for the presence or absence of a bearing (inside)
- and by sensors to determine their material

The system is controlled by a program, in two versions: with an event-driven task, triggered when the testing position is reached, or without it.

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as a network of timed automata, handling clocks and discrete variables and communicating through binary and broadcast channels.
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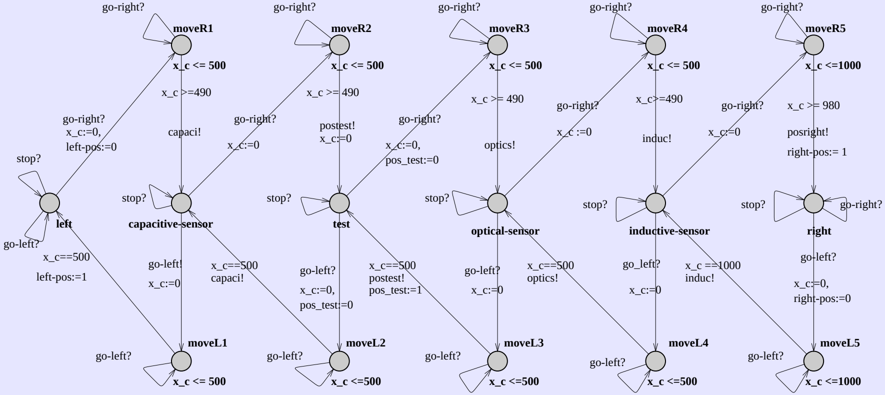
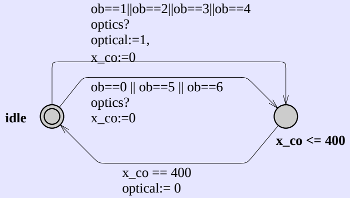
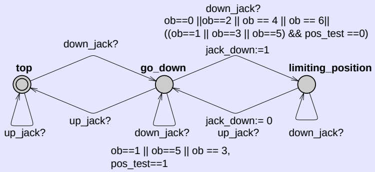


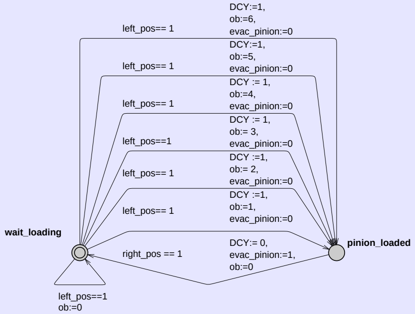
Modeling station 2 of the platform (2)

other elements

An optical sensor, the jack and the environment (abstracted):







Modeling the control program (1)

written in *Ladder Diagram* (IEC 61131-3)

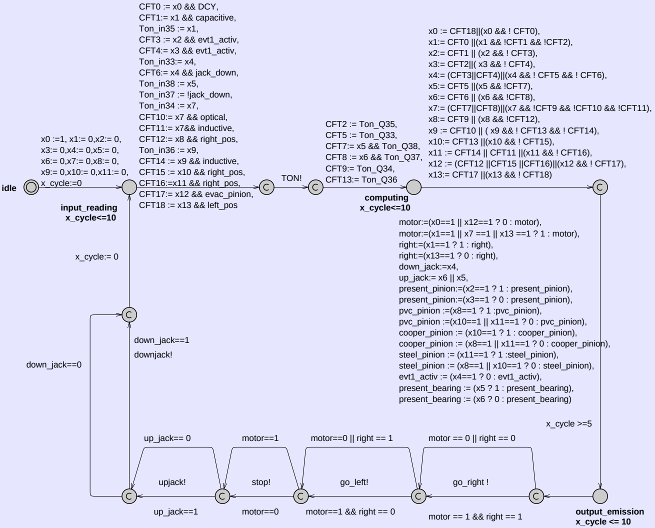
```
|     a      b      ST     |
+---| |---| |---(   )---+
|     c             MA     |
+---|/|---+-----(   )---+
|     d    |               |
+---| |---+                |
|                          |
```

means

```
ST := a and b
MA := not(c) or d
```

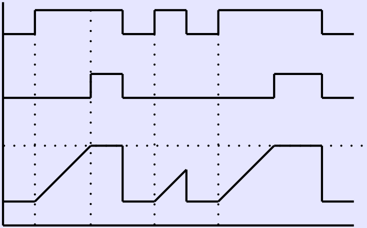
Modeling the control program (2)

in UPPAAL



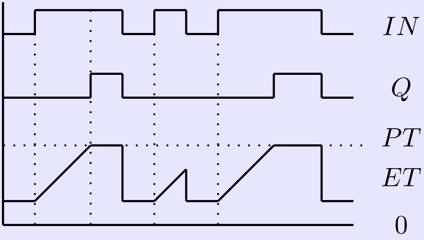
Time in PLCs

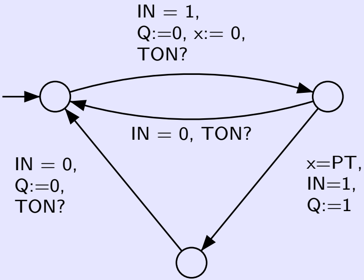
Timer On Delay (TON)



Time in PLCs

Timer On Delay (TON)





Results

Verification uses an observer automaton with clock X, reset when the signal is sent and tested when the conveyor stops.

property	result	time	memory
with the event driven task			
C1:E<> obs.stop and $X > 5$	yes	15 s	30 Mb
C2:E<> obs.stop and $X \leq 5$	yes	15 s	30 Mb
C3:E<> obs.stop and $X > 10$	no	22 s	61 Mb
without the event driven task			
C5:E<> obs.stop and $X \geq 10$	yes	16 s	30 Mb
C6:E<> obs.stop and $X > 20$	no	22 s	70 Mb
C7:E<> obs.stop and $X < 10$	no	22 s	69 Mb
with Mader-Wupper model			
C8:E<> obs.stop and $X > 5$	-	$>$29 h	-

Linux machine, pentium4 at 2.4 GHz with 3 Gb RAM

- Multitask programming reduces the reaction time from two to one cycle time.
- However, C1 proves that it is not sufficient to satisfy requirement **P**.

Performances (14 automata, 11 clocks, 30.10^6 states) are due to an atomicity hypothesis in the control program and enhanced model of the TON block.

Outline

Timed Models

Verification

Applications

Conclusion

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Many works in this area

- for other models and other logics
- for quantitative extensions with weights, costs, probabilities, etc.
- relating control problems with game theory

Perspectives

Theoretical: refine the limits for decidability questions
Practical : deal with the combinatorial explosion problem

- specifications and models fitting particular settings, with simpler and more efficient algorithms
- data structures for the combination of discrete and continuous features
- abstraction methods

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Thank you

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