

**RDBMS**

**Relational Algebra**

# Relational algebra

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- Operands: relations (tables)
- Closure: the result of any operation is another relation
- Complete: all combinations of operators allowed
- Unary operators (single operand):  
sélection (noté  $\sigma$ ), projection ( $\pi$ )
- Binary operators:  
Cartesian product ( $\times$ ), join ( $\bowtie$ ), union ( $\cup$ ), intersection ( $\cap$ ), set difference ( $-$ ), division ( $/$ )

# Outline

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- For each of these 8 operators:
  - ◆ the operation
  - ◆ syntax (notation)
  - ◆ semantics (expected result)
  - ◆ schema
  - ◆ some annotation
  - ◆ an example

# Selection

$\sigma$

- Goal: only select some tuples (lines) of a relation

| Country | name    | capital | population | surface |
|---------|---------|---------|------------|---------|
|         | Austria | Vienna  | 8          | 83      |
|         | UK      | London  | 56         | 244     |
|         | Switz.  | Berne   | 7          | 41      |

We wish to select only countries with a small surface :

**small-country =  $\sigma$  [surface < 100] Country**

| <b>small-Country</b> | name          | capital           | population    | surface        |
|----------------------|---------------|-------------------|---------------|----------------|
|                      | Austria       | Vienna            | 8             | 83             |
|                      | <del>UK</del> | <del>London</del> | <del>56</del> | <del>244</del> |
|                      | Switz.        | Berne             | 7             | 41             |

# Projection

$\pi$

- Goal: only keep some attributes (columns) of a relation

| Country | name    | capital | population | surface |
|---------|---------|---------|------------|---------|
|         | Austria | Vienna  | 8          | 83      |
|         | UK      | London  | 56         | 244     |
|         | Switz.  | Berne   | 7          | 41      |

We only want to keep name and capital attributes :

**capitals** =  $\pi$  [name, capital] Country

| <b>capitals</b> | name    | capital | population | surface |
|-----------------|---------|---------|------------|---------|
|                 | Austria | Vienna  | 8          | 83      |
|                 | UK      | London  | 56         | 244     |
|                 | Switz.  | Berne   | 7          | 41      |

# Side-effect of projection

- Elimination of repeated tuples

- ◆ A projection that does not preserve the primary key of a relation may produce identical tuples in its result
- ◆ The result will only contain one instance of the tuple
- ◆ In SQL, this is not the default behavior, use DISTINCT keyword to force this behavior

R (B, C, D)

$\pi$  (B, C) R

|   |   |   |
|---|---|---|
| b | c | d |
| a | a | b |
| a | a | c |

three tuples

|   |   |
|---|---|
| b | c |
| a | a |

two tuples

# Selection-projection

- We want the capitals of smalls Country:
  - ◆  $\text{small-Country} = \sigma [\text{surface} < 100] \text{Country}$
  - ◆  $\text{capitals} = \pi [\text{name}, \text{capital}] \text{small-Country}$

$\text{capital-small-Country} =$

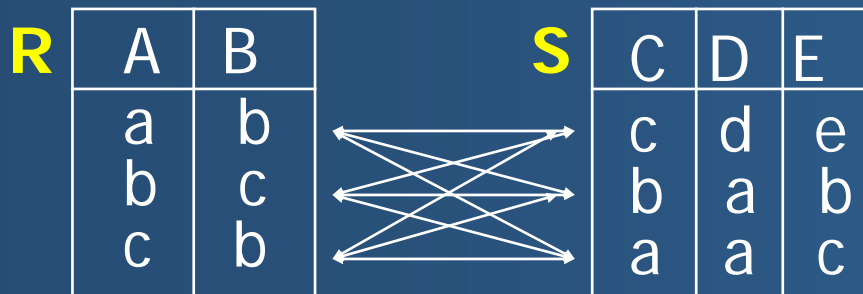
$\pi [\text{name}, \text{capital}] \sigma [\text{surface} < 100] \text{Country}$

| <u>name</u> | capital | population | surface |
|-------------|---------|------------|---------|
| Ireland     | Dublin  | 3          | 70      |
| Austria     | Vienna  | 8          | 83      |
| UK          | London  | 56         | 244     |
| Switz.      | Berne   | 7          | 41      |

(grey and beige parts eliminated) 7

# Cartesian product $\times$

- Goal: construct all combinations of tuples of two relations (usually before a selection)
- syntax :  $R \times S$
- example :



n tuples

m tuples

**$R \times S$**

| A | B | C | D | E |
|---|---|---|---|---|
| a | b | c | d | e |
| a | b | b | a | b |
| a | b | a | a | c |
| b | c | c | d | e |
| b | c | b | a | b |
| b | c | a | a | c |
| c | b | c | d | e |
| c | b | b | a | b |
| c | b | a | a | c |

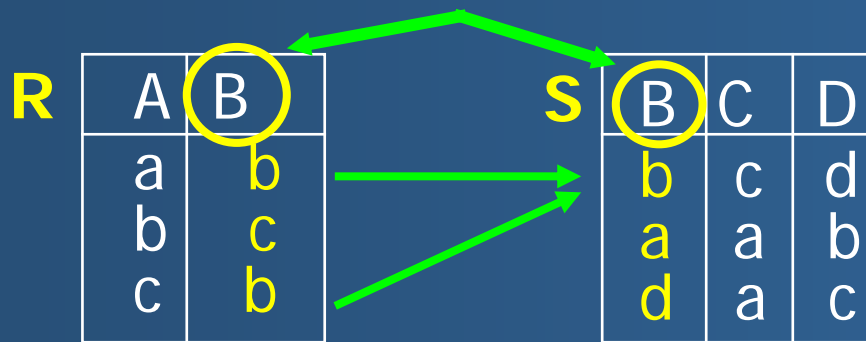
n x m tuples



# Natural join



- Goal: create all **significant** combinations of the tuples of two relations
  - ◆ significant = bear the same value for the attribute on which the join is performed
- precondition: the two relations have an attribute of a the same type
- example :



$R \bowtie S$

| A | B | C | D |
|---|---|---|---|
| a | b | c | d |
| c | b | c | d |

$$R.B = S.B$$

# Union



- binary operator
- syntax :  $R \cup S$
- semantics : adds into a single relation the tuples (lines) of R and S
- schema :  $\text{schema}(R \cup S) = \text{schema}(R) = \text{schema}(S)$
- precondition :  $\text{schema}(R) = \text{schema}(S)$
- example :

**R1**

| A | B |
|---|---|
| a | b |
| b | b |
| y | z |

**R2**

| A | B |
|---|---|
| u | v |
| y | z |

**R1  $\cup$  R2**

| A | B |
|---|---|
| a | b |
| b | b |
| y | z |
| u | v |

# Intersection $\cap$

- binary operator
- syntax :  $R \cap S$
- semantics : selects tuples that belong to both R and S
- schema :  $\text{schema}(R \cap S) = \text{schema}(R) = \text{schema}(S)$
- precondition :  $\text{schema}(R) = \text{schema}(S)$
- example :

**R1**

| A | B |
|---|---|
| a | b |
| y | z |
| b | b |

**R2**

| A | B |
|---|---|
| u | v |
| y | z |

**R1  $\cap$  R2**

| A | B |
|---|---|
| y | z |

# Set Difference -

- binary operator
- syntax :  $R - S$
- semantics : selects tuples of  $R$  that are not in  $S$
- schema :  $\text{schema}(R - S) = \text{schema}(R) = \text{schema}(S)$
- precondition :  $\text{schema}(R) = \text{schema}(S)$
- example :

**R1**

| A | B |
|---|---|
| a | b |
| y | z |
| b | b |

**R2**

| A | B |
|---|---|
| u | v |
| y | z |

**R1 - R2**

| A | B |
|---|---|
| a | b |
| b | b |

# Division /

- Goal: treat requests of the type «the ... such that ALL the...»
- let  $R(A_1, \dots, A_n)$  and  $V(A_1, \dots, A_m)$  with  $n > m$  and  $A_1, \dots, A_m$  attributes of the same name in  $R$  and  $V$
- $R/V = \{ \langle a_{m+1}, a_{m+2}, \dots, a_n \rangle / \forall \langle a_1, a_2, \dots, a_m \rangle \in V, \exists \langle a_1, a_2, \dots, a_m, a_{m+1}, a_{m+2}, \dots, a_n \rangle \in R \}$

examples : **R**

| A | B | C |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 0 |
| 1 | 2 | 1 |
| 1 | 3 | 0 |
| 2 | 1 | 1 |
| 2 | 3 | 3 |
| 3 | 1 | 1 |
| 3 | 2 | 0 |
| 3 | 2 | 1 |

| <b>V</b> | <table border="1"><tr><th>B</th><th>C</th></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>0</td></tr></table> | B | C | 1 | 1 | 2 | 0 | <b>R/V</b> | <table border="1"><tr><th>A</th></tr><tr><td>1</td></tr><tr><td>3</td></tr></table> | A | 1 | 3 |
|----------|---|---|---|---|---|---|---|------------|---|---|---|---|
| B        | C   |   |   |   |   |   |   |            |   |   |   |   |
| 1        | 1   |   |   |   |   |   |   |            |   |   |   |   |
| 2        | 0   |   |   |   |   |   |   |            |   |   |   |   |
| A        |   |   |   |   |   |   |   |            |   |   |   |   |
| 1        |   |   |   |   |   |   |   |            |   |   |   |   |
| 3        |   |   |   |   |   |   |   |            |   |   |   |   |

| <b>V'</b> | <table border="1"><tr><th>B</th><th>C</th></tr><tr><td>1</td><td>1</td></tr></table> | B | C | 1 | 1 | <b>R/V'</b> | <table border="1"><tr><th>A</th></tr><tr><td>1</td></tr><tr><td>2</td></tr><tr><td>3</td></tr></table> | A | 1 | 2 | 3 |
|-----------|--|---|---|---|---|-------------|--|---|---|---|---|
| B         | C  |   |   |   |   |             |  |   |   |   |   |
| 1         | 1  |   |   |   |   |             |  |   |   |   |   |
| A         |  |   |   |   |   |             |  |   |   |   |   |
| 1         |  |   |   |   |   |             |  |   |   |   |   |
| 2         |  |   |   |   |   |             |  |   |   |   |   |
| 3         |  |   |   |   |   |             |  |   |   |   |   |

| <b>V''</b> | <table border="1"><tr><th>B</th><th>C</th></tr><tr><td>3</td><td>5</td></tr></table> | B | C | 3 | 5 | <b>R/V''</b> | <table border="1"><tr><th>A</th></tr><tr><td>/</td></tr></table> | A | / |
|------------|--|---|---|---|---|--------------|--|---|---|
| B          | C  |   |   |   |   |              |  |   |   |
| 3          | 5  |   |   |   |   |              |  |   |   |
| A          |  |   |   |   |   |              |  |   |   |
| /          |  |   |   |   |   |              |  |   |   |

# example division

■ R

| STUDENT  | COURSE | PASSED |
|----------|--------|--------|
| Francois | RDB    | yes    |
| Francois | Prog   | yes    |
| Jacques  | RDB    | yes    |
| Jacques  | Math   | yes    |
| Pierre   | Prog   | yes    |
| Pierre   | RDB    | no     |

V

| COURSE | PASSED |
|--------|--------|
| Prog   | yes    |
| RDB    | yes    |

R/V

| STUDENT  |
|----------|
| Francois |

# Division

| certifications | PILOTE | APPAREIL |
|----------------|--------|----------|
|                | Sierra | 737      |
|                | Sierra | 757      |
|                | Sierra | 747      |
|                | Delta  | 320      |
|                | Delta  | 757      |
|                | Alpha  | 737      |
|                | Alpha  | 757      |
|                | Alpha  | 747      |
|                | Alpha  | 320      |
|                | India  | 737      |

| avions | APPAREIL |
|--------|----------|
|        | 320      |

certificationsA = certifications ÷ avions

| certificationsA | PILOTE |
|-----------------|--------|
|                 | Delta  |
|                 | Alpha  |

# Division

| certifications | PILOTE | APPAREIL |
|----------------|--------|----------|
|                | Sierra | 737      |
|                | Sierra | 757      |
|                | Sierra | 747      |
|                | Delta  | 320      |
|                | Delta  | 757      |
|                | Alpha  | 737      |
|                | Alpha  | 757      |
|                | Alpha  | 747      |
|                | Alpha  | 320      |
|                | India  | 737      |

| avions | APPAREIL |
|--------|----------|
|        | 737      |
|        | 757      |
|        | 747      |

certificationsA = certifications ÷ avions

| certificationsA | PILOTE |
|-----------------|--------|
|                 | Sierra |
|                 | Alpha  |



# Examples of algebraic requests

- let us consider the following relations :

Journal (code-j, title, price, type, periodicity)

Depot (no-Depot, name-Depot, adress)

Delivery (no-Depot, code-j, date-deliv, quantity-delivered)

# Satisfy these requests :

- What is the price of the journals ?  
 $\pi$  [price] Journal
- Give all known information on weekly journals.  
 $\sigma$  [periodicity = "weekly"] Journal
- Give the codes of the journals delivered in Paris.  
 $\pi$  [code-j] (  $\sigma$  [adress = "Paris"] Depot  $\bowtie$  Delivery)

# Satisfy these requests :

- Give the number of the depots that receive several journals.

$\pi$  [no-Depot]

( $\sigma$  [code-j  $\neq$  code' ]

( $\pi$  [no-Depot, code' ]  $\alpha$  [code-j, code' ] Delivery)

$\bowtie$   $\pi$  [no-Depot, code-j] Delivery)

$\pi$  Note :  $\alpha$  [code-j, code' ] renames attribute code-j into code'

$\pi$  Algebraic trees allow to reason on request evaluation order and request optimization

# Give the number of the depots that receive several journals :

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